We have seen that the b2 - 4ac portion of the quadratic formula, called the discriminant, can tell us the type of roots of a quadratic equation. The quadratic formula can also give us information about the relationship between the roots and the coefficient of the second term and the constant of the equation itself. Consider the following: Given a quadratic equation: ax2 + bx + c = 0 By the quadratic formulas, the two roots can be represented as Sum of the Roots, r1 + r2: Product of the Roots, r1 • r2: The sum of the roots of a quadratic equation is equal to the negation of the coefficient of the second term, divided by the leading coefficient. The product of the roots of a quadratic equation is equal to the constant term (the third term), divided by the leading coefficient. You will discover in future courses, that these types of relationships also extend to equations of higher degrees. We can also see these relationships emerge from factoring the quadratic equation. The roots will be represented as r1 and r2. Starting with ax2 + bx + c = 0; a ≠ 0 Create a leading coefficient of 1: Express the factors of the equation: Multiply and compare the equations: Conclusion: Let's put this new information to work: Yes, this question can be answered by simply multiplying the factors formed by these roots: But, let's put our new formulas to use and apply the relationship between the roots and the coefficients and constants. The sum of the roots is So the coefficient of the second term is The product of the roots is NOTE: The re-posting of materials (in part or whole) from this site to the Internet is copyright violation and is not considered "fair use" for educators. Please read the "Terms of Use". Share — copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt — remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution — You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike — If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. 100%(6)100% found this document useful (6 votes)23K viewsSaveSave MODULE 1B For Later100%100% found this document useful, undefined A quadratic equation has no real solution if the value of the discriminant is negative.When we find the roots of a quadratic equation, we usually come across one or two real solutions, but it is also possible that we don't get any real solutions. In this article, we will discuss quadratic equations in detail and what happens when they don't have real solutions, along with numerical examples.There are three different ways to tell whether the solution to a given quadratic equation is real or not, and these methods are calculating the discriminant, looking at the graph, and looking at the coefficients.Calculating the DiscriminantThe easiest way to tell that the given quadratic equation or function has no real roots is to calculate the value of the discriminant. If it is negative, then the quadratic equation does not have any real solutions. If the quadratic equation is given as $ax^{2}+bx +c = 0$, then we can write the standard form of the quadratic formula as:$x = \dfrac{-b \pm \sqrt{b^{2}-4ac }}{2a}$In this formula, the term $b^{2}- 4ac$ is called discriminant, denoting it as "D". The quadratic equation can have three solutions depending on the value of "D".1. The solution is real if "D" is > 0. This means we have two distinct solutions.2. If "D" is equal to zero, then we have a single real solution.3. If "D" < 0, we will have two complex solutions. In this case, we do not get a real solution.So, for a quadratic equation with complex solutions, the value of $b^{2}-4ac$ will be less than zero or $b^{2}< 4ac$. Let us compare examples for each case of the discriminant.$x^{2}+ 3x + 5$$x^{2}-2x + 1$$x^{2}-3x + 2$$a = 1$, $b = 3$ and $c = 5$$a = 1$, $b = -2$ and $c = 1$$a = 1$, $b = -3$ and $c = 2$$b^{2}= 3^{2}= 9$$b^{2}= (-2)^{2}= 4$$b^{2}= (-3)^{2}= 9$$4ac = 4(1)(4) = 20$4ac = 4(1)(1) = 44ac = 4(1)(2) = 8$b^{2}< 4ac$$b^{2}= 4ac$ and $D = 0$$b^{2}> 4ac$ and $D > 0$Hence, this quadratic equation has complex roots.Hence, this quadratic equation has one real root.Hence, this quadratic equation will have two real roots.The roots of the equation are $x = -1.5 + 1.6658i$ and $-1.5 – 1.6658i$The root of the equation is $x =1$The roots of the equation are $x = 2,1$You can verify these solutions by putting the values of a, b, and c in the quadratic formula. From the above table, we can deduce that whenever $b^{2}< 4ac$, we will only get complex roots.Looking at the GraphThe second method to tell whether the quadratic equation or function has any real solution or not is by looking at the graph of the function or equation. The graph of any quadratic equation will be a parabola or bell-shaped, and we know the most important feature of a parabola is its vertex.The shape of the vertex of the parabola depends upon "a"; if the value of "a" is negative, then the shape of the vertex is like a mountain top or peak. If the value of "a" is positive, then the shape is like a valley bottom at the bottom of the mountain. A quadratic equation graph with complex solutions will not touch the x-axis.The parabola can be completely above or below the x-axis if the equation has complex solutions. When the value of $a0$, the parabola will be above the x-axis. Let us draw the graph for three equations discussed in the previous section.For the equation $x^{2}+ 3x + 5$, we know all solutions are complex, and as we can see below, the graph is above the x-axis as "a" is greater than zero. The graph is not touching the x-axis, so if you are provided with a graph and you are asked to tell whether the function has real solutions or not, you can instantly tell if the graph is not touching the x-axis then it will only have complex solutions.For the equation $x^{2}-2x +1$, we know the value of the discriminant is equal to zero; for this case, the parabola peak will always touch the x-axis. It will not go across the x-axis; the peak will land on the x-axis, as shown in the figure below.For the equation $x^{2}-3x +2$, we know the value of the discriminant is greater than zero; for this case, the parabola peak will cross the x-axis. If the value of $a > 0$, then the peak value or mountain top will be going down the x-axis and if the value of $a < 0$, then the peak value or mountain top will be above the x-axis. We show the graph below.Looking at the CoefficientsIn the third method, we look at the coefficients of the given equation. Remember the equation should be given in the normal quadratic equation form as $ax^{2}+bx + c = 0$.We can only use this method in special circumstances, for example, when we are not provided with the value of "b" or the value of "b" is equal to zero. Furthermore, the sign of the coefficients "a" and "c" must also be the same. For $b = 0$, if both "c" and "a" are positive then $\dfrac{c}{a}$ is positive and -\dfrac{c}{a} is negative and similarly if both "c" and "a" are negative then $\dfrac{c}{a}$ is positive and $-\dfrac{c}{a}$ is negative. In both cases, taking the square root will give us two complex solutions.Let us take an example of the quadratic equation $x^{2}+ 6 = 0$, we can see that in this equation $a = 1$, $b = 0$ and $c = 6$. The roots for given equation are $2.449i$ and $-2.449i$.Similarly, if we take the example of quadratic equation $-3x^{2}- 6 = 0$, we can see that in this equation $a = -3$, $b = 0$ and $c = -6$. The roots for the given equations are $1.41i$ and $-1.41i$. So, we can see that when signs of coefficients "a" and "c" were the same and b was equal to zero, we only get complex solutions.Does the Quadratic Equation Always Have a Solution?Yes, the quadratic equation will always have a solution that can either be complex or real. The quadratic equation can have a maximum of 2 real solutions. So the real solution for a quadratic equation can be 0,1, or 2, depending upon the type of quadratic equation. Similarly, the quadratic equations' complex roots can be 2 or zero. We can summarize the roots of the quadratic equation as follows:• When the value of the discriminant is positive, then we will have two real solutions.• When the value of the discriminant is equal to zero, we will have a single real solution.• When the value of the discriminant is negative, we will have two complex solutions.Examples of Quadratic EquationsLet us now study examples by solving quadratic equations having real or complex solutions. We will study no real solution quadratic equation examples and real solution quadratic equation examples.Example 1: Solve the quadratic equation $x^{2}+ 2x + 2$Solution:We know for the given quadratic equation the value of $a =1$, $b = 2$ and $c =24$The value of $b^{2}= 2^{2}= 4$$4ac = 4 (1)(2) = 8$$b^{2}- 4ac = 4 – 8 = -4$.As the value of discriminant is less than zero, then this equation will only have complex solutions. Let us put the value of a, b and c in quadratic formula and solve for the roots to verify.$x = \dfrac{-2 \pm \sqrt{-4 }}{2(1)}$$x = -1 \pm 1i$Example 2: Will the quadratic equation $-2x^{2}+4 = 0$ have real roots or not?Solution:We know for the given quadratic equation the value of $a = -2$, $b = 0$ and $c =4$.We have studied that if a quadratic equation does not have the coefficient "b" or the value of "b" is equal to zero and the sign of coefficient "a" and "b" are the same as well, then it will not have a real solution. But in this case, the sign of "a" and "b" are opposite, so this equation should have real roots.$b = 0$$4ac = 4 (-2)(4) = -32$$b^{2}- 4ac = 0 – (-32) = 32$.As the value of the discriminant is positive, it is the second indicator that tells us that this quadratic equation will have real roots. Let us put the value of a, b, and c in the quadratic formula and solve for the roots to verify.$x = \pm\dfrac{ \sqrt{32 }}{2(-2)}$$x = \pm \sqrt{2}$Hence, we have proved the equation has real roots.Example 3: Will the quadratic equation $-2x^{2}- 4 = 0$ have real roots or not?Solution:We can tell by just looking at the equation that it is a no real roots.We know for the given quadratic equation the value of $a = -2$, $b = 0$ and $c = – 2$.As discussed earlier, if the value of $b = 0$ and "a" and "b" have same sign, then there will be no real roots for the given equation and this equation fulfills all the criteria.$b = 0$$4ac = 4 (-2)(-4) = 32$$b^{2}- 4ac = 0 – (32) = -32$.As the value of the discriminant is negative, it is the second indicator that this quadratic equation will not have real roots. Let us put the value of a, b and c in the quadratic formula and solve for the roots to verify.$x = \pm\dfrac{ \sqrt{-32 }}{2(-2)}$$x = \pm \sqrt{2}i$Hence proved the equation has no real rootsExample 4: Solve the quadratic equation $x^{2}+ 5x + 10 = 0$Solution:We know for the given quadratic equation the value of $a =1$, $b = 5$ and $c = 10$The value of $b^{2}= 5^{2}= 25$$4ac = 4 (1)(10) = 40$$b^{2}- 4ac = 25 – 40 = -15$.As the value of discriminant is less than zero, then this equation will not have any real solutions. Let us put the value of a, b and c in quadratic formula and solve for the roots to verify.$x = \dfrac{-5 \pm \sqrt{-15 }}{2(1)}$$x = -2.5 \pm 1.934i$You can verify your answer quickly by using a no-real solution calculator online.How To Write a Quadratic Equation Using the Complex RootsIt is quite easy to write a quadratic equation if you are provided with the complex roots. Suppose we are given the roots of the equation as $4i$ and $-4i$ and we are asked to find the original quadratic equation. We can do that by using the formula $(x-a) (x-b)$ let $a = 4i$ and $b = -4i$.$(x- 4i) (x-(-4i)$$(x-4i) (x+4i)$$x^{2}- 16i^{2}$$x^{2}-16(-1) = x^{2}+ 16$. So the quadratic equation for roots $4i$ and $-4i$ is $x^{2} +16$.Frequently Asked QuestionsWhat Is a Real Solution?A real solution is a solution to an equation that only contains real numbers. In literature, you will often learn that if a quadratic equation discriminant is less than zero, it does not have a solution. It means that it does not have a real solution.What Is a Non-real Solution?A solution that contains imaginary numbers or is written in the form $a+bi$ is called a non-real or complex solution. Here, "a" is real, and the coefficient "b" has iota attached to it, which makes the term imaginary.How Can a Quadratic Equation Have No Solution?The quadratic equation will always have a solution. It will either be real or complex, but there will always be roots for the equation.ConclusionLet us conclude our topic discussion and summarize what we have learned so far.• Quadratic equation will always have a solution, and it can either be real or complex depending upon the value of the discriminant.• There will be no real roots if the value of discriminant is less than zero or $b^{2}-4ac < 0$ or $b^{2} < 4ac$.• When the value of the discriminant is less than zero, we will have two complex solutions and no real rootsAfter studying this guide, we hope you can quickly identify when a quadratic has real solutions and when it only has complex solutions. 0 ratings0% found this document useful (0 votes)9K viewsThis document contains answer keys for a math module on quadratic equations. It provides the answers to various practice problems in the module, identifying whether equations are quadratic o…AI-enhanced title and descriptionSaveSave Math 9_Q1_M1_Answer Key For Later0%0% found this document useful, undefined Polynomial equation of degree two In mathematics, a quadratic equation (from Latin quadratus 'square') is an equation that can be rearranged in standard form as[1] a x 2 + b x + c = 0 , {\displaystyle ax^{2}+bx+c=0\,,} where the variable x represents an unknown number, and a, b, and c represent known numbers, where a ≠ 0. (If a = 0 and b ≠ 0 then the equation is linear, not quadratic.) The numbers a, b, and c are the coefficients of the equation and may be distinguished by respectively calling them, the quadratic coefficient, the linear coefficient and the constant coefficient or free term.[2] The values of x that satisfy the equation are called solutions of the equation, and roots or zeros of the quadratic function on its left-hand side. A quadratic equation has at most two solutions. If there is only one solution, one says that it is a double root. If all the coefficients are real numbers, there are either two real solutions, or a single real double root, or two complex solutions that are complex conjugates of each other. A quadratic equation always has two roots, if complex roots are included and a double root is counted for two. A quadratic equation can be factored into an equivalent equation[3] a x 2 + b x + c = a (x − r) (x − s) = 0 {\displaystyle ax^{2}+bx+c=a(x-r)(x-s)=0} where r and s are the solutions for x. The quadratic formula x = − b ± b 2 − 4 a c 2 a {\displaystyle x={\frac {-b\pm {\sqrt {b^{2}-4ac}}}{2a}}} expresses the solutions in terms of a, b, and c. Completing the square is one of several ways for deriving the formula. Solutions to problems that can be expressed in terms of quadratic equations were known as early as 2000 BC.[4][5] Because the quadratic equation involves only one unknown, it is called "univariate". The quadratic equation contains only powers of x that are non-negative integers, and therefore it is a polynomial equation. In particular, it is a second-degree polynomial equation, since the greatest power is two. Figure 1. Plots of quadratic function y = ax2 + bx + c, varying each coefficient separately while the other coefficients are fixed (at values a = 1, b = 0, c = 0) A quadratic equation whose coefficients are real numbers can have either zero, one, or two distinct real-valued solutions, also called roots. When there is only one distinct root, it can be interpreted as two roots with the same value, called a double root. When there are no real roots, the coefficients can be considered as complex numbers with zero imaginary part, and the quadratic equation still has two complex-valued roots, complex conjugates of each-other with a non-zero imaginary part. A quadratic equation whose coefficients are arbitrary complex numbers always has two complex-valued roots which may or may not be distinct. The solutions of a quadratic equation can be found by several alternative methods. It may be possible to express a quadratic equation ax2 + bx + c = 0 as a product (px + q)(rx + s) = 0. In some cases, it is possible, by simple inspection, to determine values of p, q, r, and s that make the two forms equivalent to one another. If the quadratic equation is written in the second form, then the "Zero Factor Property" states that the quadratic equation is satisfied if px + q = 0 or rx + s = 0. Solving these two linear equations provides the roots of the quadratic. For most students, factoring by inspection is the first method of solving quadratic equations to which they are exposed.[6]: 202–207 If one is given a quadratic equation in the form x2 + bx + c = 0, the sought factorization has the form (x + q)(x + s), and one has to find two numbers q and s that add up to b and whose product is c (this is sometimes called "Vieta's rule"[7] and is related to Vieta's formulas). As an example, x2 + 5x + 6 factors as (x + 3)(x + 2). The more general case where a does not equal 1 can require a considerable effort in trial and error guess-and-check, assuming that it can be factored at all by inspection. Except for special cases such as where b = 0 or c = 0, factoring by inspection only works for quadratic equations that have rational roots. This means that the great majority of quadratic equations that arise in practical applications cannot be solved by factoring by inspection.[6]: 207 Main article: Completing the square Figure 2. For the quadratic function y = x2 − x − 2, the points where the graph crosses the x-axis, x = −1 and x = 2, are the solutions of the quadratic equation x2 − x − 2 = 0. The process of completing the square makes use of the algebraic identity x 2 + 2 h x + h 2 = (x + h) 2 , {\displaystyle x^{2}+2hx+h^{2}=(x+h)^{2},} which represents a well-defined algorithm that can be used to solve any quadratic equation.[6]: 207 Starting with a quadratic equation in standard form, ax2 + bx + c = 0 Divide each side by a, the coefficient of the squared term. Subtract the constant term c/a from both sides. Add the square of one-half of b/a, the coefficient of x, to both sides. This "completes the square", converting the left side into a perfect square. Write the left side as a square and simplify the right side if necessary. Produce two linear equations by equating the square root of the left side with the positive and negative square roots of the right side. Solve each of the two linear equations. We illustrate use of this algorithm by solving 2x2 + 4x − 4 = 0 2 x 2 + 4 x − 4 = 0 {\displaystyle 2x^{2}+4x-4=0} x 2 + 2 x − 2 = 0 {\displaystyle \ x^{2}+2x-2=0} x 2 + 2 x = 2 {\displaystyle \ x^{2}+2x=2} x 2 + 2 x + 1 = 2 + 1 {\displaystyle \ x^{2}+2x+1=2+1} (x + 1) 2 = 3 {\displaystyle \left(x+1\right)^{2}=3} x + 1 = ± 3 {\displaystyle \ x+1=\pm {\sqrt {3}}} x = − 1 ± 3 {\displaystyle \ x=-1\pm {\sqrt {3}}} The plus–minus symbol "±" indicates that both x = − 1 + 3 {\textstyle x=-1+{\sqrt {3}}} and x = − 1 − 3 {\textstyle x=-1-{\sqrt {3}}} are solutions of the quadratic equation.[8] Main article: Quadratic formula Completing the square can be used to derive a general formula for solving quadratic equations, called the quadratic formula.[9] The mathematical proof will now be briefly summarized.[10] It can easily be seen, by polynomial expansion, that the following equation is equivalent to the quadratic equation: (x + b 2 a) 2 = b 2 − 4 a c 4 a 2 . {\displaystyle \left(x+{\frac {b}{2a}}\right)^{2}={\frac {b^{2}-4ac}{4a^{2}}}.} Taking the square root of both sides, and isolating x, gives: x = − b ± b 2 − 4 a c 2 a . {\displaystyle x={\frac {-b\pm {\sqrt {b^{2}-4ac}}}{2a}}.} Some sources, particularly older ones, use alternative parameterizations of the quadratic equation such as ax2 + 2bx + c = 0 or ax2 − 2bx + c = 0 ,[11] where b has a magnitude one half of the more common one, possibly with opposite sign. These result in slightly different forms for the solution, but are otherwise equivalent. A number of alternative derivations can be found in the literature. These proofs are simpler than the standard completing the square method, represent interesting applications of other frequently used techniques in algebra, or offer insight into other areas of mathematics. A lesser known quadratic formula, as used in Muller's method, provides the same roots via the equation x = 2 c − b ± b 2 − 4 a c . {\displaystyle x={\frac {2c}{-b\pm {\sqrt {b^{2}-4ac}}}}.} This can be deduced from the standard quadratic formula by Vieta's formulas, which assert that the product of the roots is c/a. It also follows from dividing the quadratic equation by x 2 {\displaystyle x^{2}} giving c x − 2 + b x − 1 + a = 0 , {\displaystyle cx^{-2}+bx^{-1}+a=0,} solving this for x − 1 , {\displaystyle x^{-1},} and then inverting. One property of this form is that it yields one valid root when a = 0, while the other root contains division by zero, because when a = 0, the quadratic equation becomes a linear equation, which has one root. By contrast, in this case, the more common formula has a division by zero for one root and an indeterminate form 0/0 for the other root. On the other hand, when c = 0, the more common formula yields two correct roots whereas this form yields the zero root and an indeterminate form 0/0. When neither a nor c is zero, the equality between the standard quadratic formula and Muller's method, 2 c − b − b 2 − 4 a c = − b + b 2 − 4 a c 2 a , {\displaystyle {\frac {2c}{-b-{\sqrt {b^{2}-4ac}}}}={\frac {-b+{\sqrt {b^{2}-4ac}}}{2a}}\,,} can be verified by cross multiplication, and similarly for the other choice of signs. It is sometimes convenient to reduce a quadratic equation so that its leading coefficient is one. This is done by dividing both sides by a, which is always possible since a is non-zero. This produces the reduced quadratic equation:[12] x 2 + p x + q = 0 , {\displaystyle x^{2}+px+q=0,} where p = b/a and q = c/a. This monic polynomial equation has the same solutions as the original. The quadratic formula for the solutions of the reduced quadratic equation, written in terms of its coefficients, is x = − p 2 ± (p 2) 2 − q . {\displaystyle x=-{\frac {p}{2}}\pm {\sqrt {\left({\frac {p}{2}}\right)^{2}-q}}\,.} Figure 3. Discriminant signs In the quadratic formula, the expression underneath the square root sign is called the discriminant of the quadratic equation, and is often represented using an upper case D or an upper case Greek delta:[13] Δ = b 2 − 4 a c . {\displaystyle \Delta =b^{2}-4ac.} A quadratic equation with real coefficients can have either one or two distinct real roots, or two distinct complex roots. In this case the discriminant determines the number and nature of the roots. There are three cases: If the discriminant is positive, then there are two distinct roots − b + Δ 2 a and − b − Δ 2 a , {\displaystyle {\frac {-b+{\sqrt {\Delta }}}{2a}}\quad {\text{and}}\quad {\frac {-b-{\sqrt {\Delta }}}{2a}},} both of which are real numbers. For quadratic equations with rational coefficients, if the discriminant is a square number, then the roots are rational—in other cases they may be quadratic irrationals. If the discriminant is zero, then there is exactly one real root − b 2 a , {\displaystyle -{\frac {b}{2a}},} sometimes called a repeated or double root or two equal roots. If the discriminant is negative, then there are no real roots. Rather, there are two distinct (non-real) complex roots[14] − b 2 a + i − Δ 2 a and − b 2 a − i − Δ 2 a , {\displaystyle -{\frac {b}{2a}}+i{\frac {\sqrt {-\Delta }}{2a}}\quad {\text{and}}\quad -{\frac {b}{2a}}-i{\frac {\sqrt {-\Delta }}{2a}},} which are complex conjugates of each other. In these expressions i is the imaginary unit. Thus the roots are distinct if and only if the discriminant is non-zero, and the roots are real if and only if the discriminant is non-negative. Visualisation of the complex roots of y = ax2 + bx + c: the parabola is rotated 180° about its vertex (orange). Its x-intercepts are rotated 90° around their mid-point, and the Cartesian plane is interpreted as the complex plane (green).[15] The function f(x) = ax2 + bx + c is a quadratic function.[16] The graph of any quadratic function has the same general shape, which is called a parabola. The location and size of the parabola, and how it opens, depend on the values of a, b, and c. If a > 0, the parabola has a minimum point and opens upward. If a < 0, the parabola has a maximum point and opens downward. The extreme point of the parabola, whether minimum or maximum, corresponds to its vertex. The x-coordinate of the vertex will be located at x = − b 2 a {\displaystyle \scriptstyle x={\tfrac {-b}{2a}}} , and the y-coordinate of the vertex may be found by substituting this x-value into the function. The y-intercept is located at the point (0, c). The solutions of the quadratic equation ax2 + bx + c = 0 correspond to the roots of the function f(x) = ax2 + bx + c, since they are the values of x for which f(x) = 0. If a, b, and c are real numbers and the domain of f is the set of real numbers, then the roots of f are exactly the x-coordinates of the points where the graph touches the x-axis. If the discriminant is positive, the graph touches the x-axis at two points; if zero, the graph touches at one point; and if negative, the graph does not touch the x-axis. The term x − r {\displaystyle x-r} is a factor of the polynomial a x 2 + b x + c {\displaystyle ax^{2}+bx+c} if and only if r is a root of the quadratic equation a x 2 + b x + c = 0. {\displaystyle ax^{2}+bx+c=0.} It follows from the quadratic formula that a x 2 + b x + c = a (x − − b + b 2 − 4 a c 2 a) (x − − b − b 2 − 4 a c 2 a) . {\displaystyle ax^{2}+bx+c=a\left(x-{\frac {-b+{\sqrt {b^{2}-4ac}}}{2a}}\right)\left(x-{\frac {-b-{\sqrt {b^{2}-4ac}}}{2a}}\right).} In the special case b2 = 4ac where the quadratic has only one distinct root (i.e. the discriminant is zero), the quadratic polynomial can be factored as a x 2 + b x + c = a (x + b 2 a) 2 . {\displaystyle ax^{2}+bx+c=a\left(x+{\frac {b}{2a}}\right)^{2}.} Figure 4. Graphing calculator computation of one of the two roots of the quadratic equation 2x2 + 4x − 4 = 0. Although the display shows only five significant figures of accuracy, the retrieved value of xc is 0.732050807569, accurate to twelve significant figures. A quadratic function without real root: y = (x − 5)2 + 9. The "3" is the imaginary part of the x-intercept. The real part is the x-coordinate of the vertex. Thus the roots are 5 ± 3i. The solutions of the quadratic equation a x 2 + b x + c = 0 {\displaystyle ax^{2}+bx+c=0} may be deduced from the graph of the quadratic function f (x) = a x 2 + b x + c , {\displaystyle f(x)=ax^{2}+bx+c,} which is a parabola. If the parabola intersects the x-axis in two points, there are two real roots, which are the x-coordinates of these two points (also called x-intercept). If the parabola is tangent to the x-axis, there is a double root, which is the x-coordinate of the contact point between the graph and parabola. If the parabola does not intersect the x-axis, there are two complex conjugate roots. Although these roots cannot be visualized on the graph, their real and imaginary parts can be.[17] Let h and k be respectively the x-coordinate and the y-coordinate of the vertex of the parabola (that is the point with maximal or minimal y-coordinate. The quadratic function may be rewritten y = a (x − h) 2 + k . {\displaystyle y=a(x-h)^{2}+k.} Let d be the distance between the point of y-coordinate 2k on the axis of the parabola, and a point on the parabola with the same y-coordinate (see the figure; there are two such points, which give the same distance, because of the symmetry of the parabola). Then the real part of the roots is h, and their imaginary part are ±d. That is, the roots are h + i d and h − i d , {\displaystyle h+id\quad {\text{and}}\quad h-id,} or in the case of the example of the figure 5 + 3 i and 5 − 3 i . {\displaystyle 5+3i\quad {\text{and}}\quad 5-3i.} Although the quadratic formula provides an exact solution, the result is not exact if real numbers are approximated during the computation, as usual in numerical analysis, where real numbers are approximated by floating point numbers (called "reals" in many programming languages). In this context, the quadratic formula is not completely stable. This occurs when the roots have different order of magnitude, or, equivalently, when b2 and b2 − 4ac are close in magnitude. In this case, the subtraction of two nearly equal numbers will cause loss of significance or catastrophic cancellation in the smaller root. To avoid this, the root that is smaller in magnitude, r, can be computed as (c / a) / R {\displaystyle (c/a)/R} where R is the root that is bigger in magnitude. This is equivalent to using the formula x = − 2 c b ± b 2 − 4 a c {\displaystyle x={\frac {-2c}{b\pm {\sqrt {b^{2}-4ac}}}}} using the plus sign if b > 0 {\displaystyle b>0} and the minus sign if b < 0. {\displaystyle b