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Linear Functions: You are already familiar with the concept of "average rate of change". When working with straight lines (linear functions) you saw the "average rate of change" to be: The word "slope" may also be referred to as "gradient", "incline" or "pitch", and be expressed as: A special circumstance exists when working with straight lines (linear functions), in that the "average rate of change" (the slope) is constant. No matter where you check the slope on a straight line, you will get the same answer. Non-Linear Functions: When working with non-linear functions, the "average rate of change" is not constant. The process of computing the "average rate of change", however, remains the same as was used with straight lines: two points are chosen, and is computed. FYI: You will learn in later courses that the "average rate of change" in non-linear functions is actually the slope of the secant line passing through the two chosen points. A secant line cuts a graph in two points. When you find the "average rate of change" you are finding the rate at which (how fast) the function's y-values (output) are changing as compared to the function's x-values (input). When working with functions (of all types), the "average rate of change" is expressed using function notation. Average Rate of Change For the function y = f (x) between x = a and x = b, the While this new formula may look strange, it is really just a re-write of . Remember that y = f (x). So, when working with points (x1, y1) and (x2, y2), we can also write them as the points . Then our slope formula can be expressed as . If we rename x1 to be a, and x2 to be b, we will have the new formula. The points are , and the . (Just remember that it is the "slope" formula.) Finding average rate of change from a table. Function f (x) is shown in the table at the right. Find the average rate of change over the interval 1 < x < 3. Solution: If the interval is 1 < x < 3, then you are examining the points from (1,4) to (3,16). From the first point, let a = 1, and f (a) = 4. From the second point, let b = 3 and f (b) = 16. Substitute into the formula: The average rate of change is 6 over 1, or just 6 over the interval 1 < x < 3. The y-values change 6 units every time the x-values change 1 unit, on this interval. Finding average rate of change from a graph. Function g (x) is shown in the graph at the right. Find the average rate of change over the interval 1 < x < 4. Solution: If the interval is 1 < x < 4, then you are examining the points (1,1) and (4,2), as seen on the graph. From the first point, let a = 1, and g (a) = 1. From the second point, let b = 4 and g (b) = 2. Substitute into the formula: The average rate of change is 1 over 3, or just 1/3 on the interval 1 < x < 4. The y-values change 1 unit every time the x-values change 3 units, on this interval. Finding average rate of change from a word problem. A ball thrown in the air has a height of $h(t) = - 16t^2 + 50t + 3$ feet after t seconds. a) What are the units of measurement for the average rate of change of h? b) Find the average rate of change of h between t = 0 and t = 1.5. c) Find the average rate of change of h between t = 2 and t = 3. Solution: a) In the formula, , the numerator (top) is measured in feet and the denominator (bottom) is measured in seconds. This ratio is measured in feet per second, which will be the velocity of the ball. b) Start by finding h(t) when t = 0 and t = 1.5, by plugging the t values into h(t). $h(1.5) = -16(1.5)^2 + 50(1.5) + 3 = 42$ $h(0) = -16(0)^2 + 50(0) + 3 = 3$ Now, use the average rate of change formula: The ball is going up (+). c) Same approach as part b. $h(3) = -16(3)^2 + 50(3) + 3 = 9$ $h(2) = -16(2)^2 + 50(2) + 3 = 39$ Now, use the average rate of change formula: The ball is coming down (-). In Example 3, we saw a rate of change problem whose results carried with them (+) or (-) signs to designate the direction of the ball (rising versus falling). Due to this designation of "direction", the answers are referred to as velocity and not speed.Unlike velocity, speed is always positive. Speed is the ratio of the distance traveled to the time interval. (Distance is always positive. Time is a always positive. Speed is a always positive.) Think about a car speedometer. The speedometer tells you the car's speed in miles per hour, and the result is always a positive number. The speedometer does not register negative numbers when the car backs up. Speed does not care about "direction". When dealing with "average rate of change", you may encounter questions that ask you to find the "average speed". Average Speed Caution: Remember, speed is always positive. Speed does not indicate direction (it is not a vector). If direction is involved, you are dealing with velocity - which is speed with a direction (displacement). (The difference between speed and velocity will be a big deal when you take Physics.) Finding average speed from a word problem. A car's starting position on a number line (representing "miles"), is located at -2. At the end of a trip, the car's position is at 132 on the number line. The trip takes 2 hours. What is the average speed of the car during this trip? Solution: NOTE: The re-posting of materials (in part or whole) from this site to the Internet is copyright violation and is not considered "fair use" for educators. Please read the "Terms of Use". Understanding the average rate of change is crucial across various fields of study, including calculus, where it represents the slope of the secant line between two points on a function. Here's a step-by-step guide to grasp this concept fully and apply it in different contexts: Definition The average rate of change is a measure of how much a quantity changes, on average, between two points. In mathematical terms, for a function \(f(x)\), the average rate of change from \(x=a\) to \(x=b\) is \(\frac{f(b)−f(a)}{b−a}\). Graphical Representation It's the slope of the straight line (secant line) connecting two points on a curve. Identify the Points Choose two points on the graph of the function or in your data set, labeled as \((a,f(a))\) and \((b,f(b))\). Apply the Formula Subtract the \(y\)-values: \(f(b)−f(a)\). Subtract the \(x\)-values: \(b−a\). Divide the difference in \(y\)-values by the difference in \(x\)-values to find the average rate of change. Positive or Negative A positive average rate of change indicates an increasing function in the interval, while a negative one indicates a decreasing function. Magnitude The greater the magnitude of the average rate of change, the steeper the line and the more significant the change over the interval. Calculus Motion: It can represent the average velocity of an object over a time interval. Functions: Helps in understanding the behavior of functions over an interval before delving into instantaneous rates of change (derivatives). Economics Market Analysis: Calculate the average rate of change of stock prices to gauge overall market trends. Growth Rates: Determine the average growth rate of a company's revenue or profit over time. Biology Population Dynamics: Measure the average growth rate of a population over a given time period. Biochemical Processes: Calculate the rate of change of reactant or product concentration in a reaction. Physics Thermodynamics: Analyze the average rate of temperature change in a substance. Kinematics: Use it to find the average acceleration when velocity changes over time. Secant Line to Tangent Line: As the interval between \(a\) and \(b\) gets smaller, the average rate of change approaches the instantaneous rate of change (the derivative). Curve Analysis: Use the average rate of change to approximate the behavior of curves before using more advanced calculus techniques. When presenting your findings, contextualize the average rate of change within the problem's framework, explaining what the change represents in real-world terms. Use graphs to illustrate the average rate of change visually for a more impactful presentation. The average rate of change is a fundamental concept that serves as a stepping stone to more advanced calculus ideas like derivatives. Its utility spans across various disciplines, making it a versatile tool for analyzing changes and trends in a myriad of contexts. By following this guide, you can harness this concept to extract meaningful insights from a range of data sets and functions. Example 1: Determine the average rate of change of the function \(g(x)=3x^2−4x+1\) from \(x=2\) to \(x=5\). Solution: Calculate \(g(2)=3(2)^2−4(2)+1=5\). Calculate \(g(5)=3(5)^2−4(5)+1=56\). Apply the average rate of change formula: \(\frac{g(5)−g(2)}{5−2}=\frac{56−5}{3}=\frac{51}{3}=17\). The average rate of change from \(x=2\) to \(x=5\) is \(17\). Example 2: Determine the average rate of change of the function \(h(x)=2x^3−3x^2+x−5\) from \(x=3\) to \(x=6\). Solution: Calculate \(h(3)=2(3)^3−3(3)^2+3−5=25\). Calculate \(h(6)=2(6)^3−3(6)^2+6−5=325\). Apply the average rate of change formula: \(\frac{h(6)−h(3)}{6-3}=\frac{325−25}{3}=\frac{300}{3}=100\). The average rate of change from \(x=3\) to \(x=6\) is \(100\). by: Effortless Math Team about 2 years ago (category: Articles) 1 year ago Related to This Article More math articles The average rate of change is defined as the average change in a function per unit over a given interval. It is the gradient of the secant line connecting the endpoints of the interval. It describes how one quantity changes with respect to another. The average rate of change is simply the gradient of the line connecting two points. For example, consider the two points 'a' and 'b' below. Point 'a' has coordinates (0, 1) and point 'b' has coordinates (4, 3). The average rate of change is simply the gradient of the red secant line shown in the diagram. We find the gradient of this line to find the average rate of change between the points. The gradient is found by dividing the change in y-coordinates by the change in x-coordinates. The change in y-coordinates between the 2 points is 2 and the change in x-coordinates between the 2 points is 4. 2 ÷ 4 = 0.5 and so, the average rate of change is 0.5. The average rate of change is a concept used in calculus to measure the gradient between two points. As the two points are brought closer together, the gradient of the secant line connecting them approaches the gradient of the tangent at the first point. In the example below, the gradient of the tangent to the curve at point 'a' is shown in green. It has a gradient of 2. The average rate of change is shown by the secant line in red. In the middle image, the gradient of the red secant line connecting points 'a' and 'b' is 0.5. In the final image, the point 'b' is brought closer to point 'a'. Here the gradient of the red secant line is now closer to the gradient of the green tangent line. As the two points get closer and closer, the gradient of the red secant line approaches the gradient of the tangent at the first point. There are two types of rate of change that can be shown on a graph: instantaneous rate of change and average rate of change. The instantaneous rate of change is equal to the gradient of the tangent to the curve at a point. This is shown by the green line in the image above. The instantaneous rate of change can be calculated by drawing a tangent at a point and calculating its gradient. Alternatively it can be calculated by differentiating the equation of the curve and then substituting in the value of x at that point. The average rate of change is equal to the gradient of the secant connecting two points. This is shown by the red line in the image above. To find the average rate of change from a graph: Find the coordinates of the two endpoints of the interval. Calculate the rise between the two points. Calculate the run between the two points. The average rate of change is equal to the rise ÷ run. For example, find the average rate of change in the interval 0 ≤ x ≤ 7 on the graph shown below. The ends of the interval have been marked by two points. Step 1. Find the coordinates of the two endpoints of the interval The first coordinate is (0, 0). The second coordinate is (7, 2) Step 2. Calculate the rise between the two points The rise is equal to the change in the y-coordinates between the two points. We go from 0 to 2 and so, the rise is equal to 2. Step 3. Calculate the run between the two points The run is equal to the change in the x-coordinates between the two points. We go from 0 to 7 and so, the run is equal to 7. Step 4. The average rate of change is equal to the rise ÷ run The rise is 2 and the run is 7. Therefore, the average rate of change is equal to 2 ÷ 7. Calculating this, this is approximately equal to 0.286. Therefore the average rate of change between (0, 0) and (7, 2) is 0.286. This means that for each 1 unit moved to the right, the graph increases by 0.286 on average in the vertical direction. The average rate of change is 0.286 and it applies to the whole region of the interval from 0 to 7. We can see in the image below that the red secant line is above the graph for almost the entire interval. Therefore it is not a good indication of the average rate of change at all points along the graph. To be a more accurate average rate of change, it would be expected that the red secant line would follow the trend of the graph more closely. The graph changes over the interval and to improve the accuracy, smaller intervals can be considered. For example, we can find the average rate of change in the interval 0 ≤ x ≤ 7. Between these points, the rise is 0.5 and the run is 5. 0.5 ÷ 5 = 0.1 and therefore the average rate of change in this interval is 0.1. This is a more accurate average rate of change for this section of the curve as the red secant line passes more closely through the middle of the curve. Now we can look at the average rate of change in the remaining section of the curve from 5 ≤ x ≤ 7. The rise is 1.5 and the run is 2. 1.5 ÷ 2 = 0.75 and so, the average rate of change in this interval is 0.75. From 0 ≤ x ≤ 5, the average rate of change is 0.1 and from 5 ≤ x ≤ 7 the average rate of change is 0.75. We can see that the graph increased much more rapidly from 5 to 7 compared to its steady rise from 0 to 5. This is why the average rate of change over the entire interval from 0 to 7 is in the middle of these values at 0.286. To find the average rate of change between two points, divide the change in y-coordinates by the change in x-coordinates over the interval. For example, find the average rate of change of in the interval . The y-coordinates of each point are found by substituting the corresponding values of x into the equation of the function. When , the y-coordinate is found by substituting into . Making the substitution, and so, the y-coordinate of this first point is equal to 1. When , the y-coordinate is found by substituting into . Making the substitution, and so, the y-coordinate of this second point is equal to 4. The average rate of change is found by dividing the change in the y-coordinates by the change in the x-coordinates. The first point has an x-coordinate of 2 and a y-coordinate of 1. The second point has an x-coordinate of 4 and a y-coordinate of 4. The change in y-coordinates is therefore 4 – 1 = 3. The change in x-coordinates is therefore 4 – 2= 2. The average rate of change is 3 ÷2 = 1.5. The average rate of change = [f(b) – f(a)]/[b – a]. This formula is used to calculate the average rate of change over the interval a≤x≤b. f(a) is the value of the y-coordinate at point a and f(b) is the value of the y-coordinate at point b. Average rate of change formula for the interval 'a' is the x-coordinate of the first pointf(a) is the y-coordinate of the first point'b' is the x-coordinate of the second pointf(b) is the y-coordinate of the second point For example between the two points (0.5, 2) and (4, 3): 'a' = 0.5f(a) = 2'b' = 4f(b) = 3 The average rate of change formula, becomes . Evaluating this formula which approximately equals 0.286. Alternate ways of writing the average rate of change formula are: To find the average rate of change from a table of values: Find the change in y-coordinate values over the interval. Find the change in x-coordinate values over the interval. Divide the change in y-coordinate values by the change in the x-coordinate values. For example, find the average rate of change in the interval from the table below. Step 1. Find the change in the y-coordinate values over the interval Over the interval from x = 3 to x = 8, the y-coordinates change from 0 to 5. Therefore the change in the y-coordinates over this interval is 5. Step 2. Find the change in x-coordinate values over the interval From x = 3 to x = 8, this is a change of 5. Step 3. Divide the change in the y-coordinate values by the change in the x-coordinate values The change in y-coordinate values is 5. The change in x-coordinate values is 5. Dividing the change in y-coordinates by the change in the x coordinates, 5 ÷ 5 = 1. Therefore the average rate of change over this interval is 1. Julia ŻuławińskaBogna Szyk and Jack Bowater536 people find this calculator helpfulThe average rate of change calculator is here to help you understand the simple concept hidden behind a long, little bit confusing name. What is the rate of change? Generally speaking, it shows the relationship between two factors. Look for a more precise average rate of change definition below. We will also demonstrate and explain the average rate of change formula with a couple of examples of how to use it. Prefer watching over reading? Learn all you need in 90 seconds with this video we made for you: Watch this on YouTube Everything keeps moving. Change is inevitable. Starting with the acceleration of your bike or car, through to population growth, from the blood flow in your veins to the symbiosis of your cells, the rate of change allows us to establish the value associated with those changes. The average rate of change is a rate that describes how one number changes, on average, in relation to another. If you have a function, it is the slope of the line drawn between two points. But don't confuse it with slope. You can use the average rate of change for any given function, not only linear ones. If you want to learn more about slope go to the slope calculator.In the following picture, we marked two points to help you better understand how to find the average rate of change.The average rate of change formula is: A = $[f(x_2) − f(x_1)] / [x_2 − x_1]$ where: $(x_1, f(x_1))$ – Coordinates of the first point; and $(x_2, f(x_2))$ – Coordinates of the second point. If it's positive, it means that one coordinate increases as the other also increases. For example, the more you ride a bike, the more calories you burn. It's equal to zero when one coordinate changes but the other one does not. A good example might be not studying for your exams. As time starts running out, the amount of things to learn doesn't change. The average rate of change is negative when one coordinate increases while the other one decreases. Let's say you're going on a vacation. The more time you spend on your travel, the closer you are to your destination.Let's calculate the average rate of change of distance (average speed) of a train going from Paris to Rome (1420.6 km). On the following chart, you can see the change in distance over time:As you see, the speed wasn't constant. The train stopped two times, and in between stops, it went significantly slower. But for calculating the average speed, the only variables that matter are the change in distance and the change in time. So, if the coordinates of the first point are (0, 0), and the coordinates of the second point are the distance between two cities, and the time of travel is (1420.6, 12.5), then: A = (1420.6 - 0) / (12.5 - 0) = 113.648 [km/h] On average, the train was going 113.648 kilometers per hour. Now, let's look at a more mathematical example.You have been given a function: $f(x) = x^2 + 5x − 7$ Find the average rate of change over the interval [-4, 6]. Find values of your function for both points: $f(x_1) = f(-4) = (-4)^2 + 5 \times (-4) − 7 = -11$ $f(x_2) = f(6) = 6^2 + 5 \times 6 − 7 = 59$ Use the average rate of change equation: A = $[f(x_2) − f(x_1)] / [x_2 − x_1]$ = [f(6) − f(-4)] / [6 − (-4)] = [59 − (-11)] / [6 − (-4)] = 70 / 10 = 7 If you enjoyed the average rate of change calculator, feel free to check out our other tools like this distance calculator, where you can find the distance between points or lines.FAQsNot precisely. The average rate of change reflects how a function changes on average between two points. On the other hand, we define the slope of a function as the slope of the line tangent to the curve at a specific point. In a linear function, every point changes identically, so the average rate of change and slope are equal.To find the average rate of change of a function, follow these steps: Get the (x, y) coordinates of the starting point. We'll call these (x_0, y_0). Get the (x, y) coordinates of the endpoint. These will be (x_1, y_1). Replace both within the average rate of change (A) formula: A = $(y_1 − y_0)/(x_1 − x_0)$. The average rate of change of y = 2x is 2. Since it is a linear function, the average rate of change is just the function's slope. In this case, for every change in the x-coordinate, the y-coordinate will double it.If the speed is constant, yes. Speed reflects how the position changes instantaneously with respect to time. So, if an object were moving at a constant speed, the average rate of change in the position would tell us at which speed it is traveling.Check out 46 similar coordinate geometry calculators blovy Published in Calculations 3 mins read Apr 21, 2025 The average change (also known as the average rate of change) is calculated using the following formula: (y2 - y1) / (x2 - x1), where (x1, y1) and (x2, y2) are two points. This formula essentially calculates the slope of the line connecting the two points. Understanding Average Change Average change measures how much a quantity changes on average over a specific interval. It's a fundamental concept with applications in various fields, including: Mathematics: Determining the slope of a secant line. Finance: Calculating the average growth rate of investments. Physics: Finding the average velocity of an object. Economics: Measuring the average change in prices or production. Formula Breakdown Let's break down the formula for average change: y2 - y1: This represents the change in the y-value (dependent variable). It's often referred to as "rise." x2 - x1: This represents the change in the x-value (independent variable). It's often referred to as "run." (y2 - y1) / (x2 - x1): This division calculates the rate at which the y-value changes per unit of change in the x-value. Steps for Calculating Average Change Identify two points: You need two data points represented as coordinates (x1, y1) and (x2, y2). These points could be given directly or extracted from a graph or table. Calculate the change in y: Subtract the initial y-value (y1) from the final y-value (y2). This gives you y2 - y1. Calculate the change in x: Subtract the initial x-value (x1) from the final x-value (x2). This gives you x2 - x1. Divide the change in y by the change in x: Divide the result from step 2 by the result from step 3. This is the average rate of change: (y2 - y1) / (x2 - x1). Example Let's say you want to find the average change in temperature between 8:00 AM and 12:00 PM. At 8:00 AM (x1), the temperature (y1) is 60°F. At 12:00 PM (x2), the temperature (y2) is 72°F. Using the formula: Average change = (72 - 60) / (12 - 8) = 12 / 4 = 3°F per hour. This means that, on average, the temperature increased by 3 degrees Fahrenheit each hour between 8:00 AM and 12:00 PM. Important Considerations Units: Always include the appropriate units for the average rate of change. In the example above, the units are degrees Fahrenheit per hour. Linearity: The average rate of change assumes a linear relationship between the two points. It doesn't necessarily reflect the exact change at every point within the interval. For a non-linear relationship, the instantaneous rate of change (using calculus) provides a more precise measure at a specific point. Context: Interpret the average rate of change within the context of the problem. Understand what the x and y values represent to make meaningful conclusions. [latex]y[/latex] 2005 2006 2007 2008 2009 2010 2011 2012 [latex]C\left(y\right)[/latex] 2.31 2.62 2.84 3.30 2.41 2.84 3.58 3.68 The price change per year is a rate of change because it describes how an output quantity changes relative to the change in the input quantity. We can see that the price of gasoline in the table above did not change by the same amount each year, so the rate of change was not constant. If we use only the beginning and ending data, we would be finding the average rate of change over the specified period of time. To find the average rate of change, we divide the change in the output value by the change in the input value. Average rate of change=[latex]\frac{\text{Change in output}}{\text{Change in input}}[/latex] =[latex]\frac{\Delta y}{\Delta x}[/latex] =[latex]\frac{{y}_{2}-{y}_{1}}{{x}_{2}-{x}_{1}}[/latex] =[latex]\frac{f\left({x}_{2}\right)-f\left({x}_{1}\right)}{{x}_{2}-{x}_{1}}[/latex] The Greek letter [latex]\Delta [/latex] (delta) signifies the change in a quantity; we read the ratio as "delta-y over delta-x" or "the change in [latex]y[/latex] divided by the change in [latex]x[/latex]." Occasionally we write [latex]\Delta f[/latex] instead of [latex]\Delta y[/latex], which still represents the change in the function's output value resulting from a change to its input value. It does not mean we are changing the function into some other function. In our example, the gasoline price increased by $1.37 from 2005 to 2012. Over 7 years, the average rate of change was [latex]\frac{\Delta y}{\Delta x}=\frac{{1.37}}{\text{7 years}}\approx 0.196\text{ dollars per year}[/latex] On average, the price of gas increased by about 19.6¢ each year. Other examples of rates of change include: A population of rats increasing by 40 rats per week A car traveling 68 miles per hour (distance traveled changes by 68 miles each hour as time passes) A car driving 27 miles per gallon (distance traveled changes by 27 miles for each gallon) The current through an electrical circuit increasing by 0.125 amperes for every volt of increased voltage The amount of money in a college account decreasing by $4,000 per quarter A rate of change describes how an output quantity changes relative to the change in the input quantity. The units on a rate of change are "output units per input units." The average rate of change between two input values is the total change of the function values (output values) divided by the change in the input values. [latex]\frac{\Delta y}{\Delta x}=\frac{f\left({x}_{2}\right)-f\left({x}_{1}\right)}{{x}_{2}-{x}_{1}}[/latex] How To: Given the value of a function at different points, calculate the average rate of change of a function for the interval between two values [latex]{x}_{1}[/latex] and [latex]{x}_{2}[/latex]. Calculate the difference [latex]{y}_{2}-{y}_{1}=\Delta y[/latex]. Calculate the difference [latex]{x}_{2}-{x}_{1}=\Delta x[/latex]. Find the ratio [latex]\frac{\Delta y}{\Delta x}[/latex]. Using the data in the table below, find the average rate of change of the price of gasoline between 2007 and 2009. [latex]y[/latex] 2005 2006 2007 2008 2009 2010 2011 2012 [latex]C\left(y\right)[/latex] 2.31 2.62 2.84 3.30 2.41 2.84 3.58 3.68 In 2007, the price of gasoline was $2.84. In 2009, the cost was $2.41. The average rate of change is [latex]\begin{cases}\frac{\Delta y}{\Delta x}=\frac{{y}_{2}-{y}_{1}}{{x}_{2}-{x}_{1}}\\ {}\\=\frac{2.41-2.84}{2009 - 2007}\\ {}\\=\frac{-0.43}{2\text{ years}}\\{} \\={-0.22}\text{ per year}\end{cases}[/latex] Note that a decrease is expressed by a negative change or "negative increase." A rate of change is negative when the output decreases as the input increases or when the output increases as the input decreases. The following video provides another example of how to find the average rate of change between two points from a table of values. Using the data in the table below, find the average rate of change between 2005 and 2010. [latex]y[/latex] 2005 2006 2007 2008 2009 2010 2011 2012 [latex]C\left(y\right)[/latex] 2.31 2.62 2.84 3.30 2.41 2.84 3.58 3.68 Solution Given the function [latex]g\left(t\right)[/latex] shown in Figure 1, find the average rate of change on the interval [latex]\left[-1,2\right][/latex]. Figure 1 Figure 2 At [latex]t=-1[/latex], the graph shows [latex]g\left(-1\right)=4[/latex]. At [latex]t=2[/latex], the graph shows [latex]g\left(2\right)=1[/latex]. The horizontal change [latex]\Delta t=3[/latex] is shown by the red arrow, and the vertical change [latex]\Delta g\left(t\right)=-3[/latex] is shown by the turquoise arrow. The output changes by –3 while the input changes by 3, giving an average rate of change of [latex]\frac{1 - 4}{2-\left(-1\right)}=\frac{-3}{3}=-1[/latex] Note that the order we choose is very important. If, for example, we use [latex]\frac{{y}_{2}-{y}_{1}}{{x}_{1}-{x}_{2}}[/latex], we will not get the correct answer. Decide which point will be 1 and which point will be 2, and keep the coordinates fixed as [latex]\left({x}_{1},{y}_{1}\right)[/latex] and [latex]\left({x}_{2},{y}_{2}\right)[/latex]. After picking up a friend who lives 10 miles away, Anna records her distance from home over time. The values are shown in the table below. Find her average speed over the first 6 hours. t (hours) 0 1 2 3 4 5 6 7 D(t) (miles) 10 55 90 153 214 240 282 300 Here, the average speed is the average rate of change. She traveled 282 miles in 6 hours, for an average speed of [latex]\begin{cases}\\ \frac{292 - 10}{6 - 0}\\ {}\\ =\frac{282}{6}\\{}\\ =47 \end{cases}[/latex] The average speed is 47 miles per hour. Because the speed is not constant, the average speed depends on the interval chosen. For the interval [2,3], the average speed is 63 miles per hour. Compute the average rate of change of [latex]f\left(x\right)={x}^{2}-\frac{1}{x}[/latex] on the interval [latex]\text{[2,}\text{4}\text{].}[/latex] We can start by computing the function values at each endpoint of the interval. [latex]\begin{cases}f\left(2\right)={2}^{2}-\frac{1}{2}& f\left(4\right)={4}^{2}-\frac{1}{4} \\ =4-\frac{1}{2} & =16-{1}{4} \\ =\frac{7}{2} & =\frac{63}{4} \end{cases}[/latex] Now we compute the average rate of change. [latex]\begin{cases}\text{Average rate of change}=\frac{f\left(4\right)-f\left(2\right)}{4 - 2}\hfill \\{}\\\text{ }=\frac{\frac{63}{4}-\frac{7}{2}}{4 - 2}\hfill \\{}\\ \text{ }\text{ }=\frac{\frac{49}{4}}{2}\hfill \\ {}\\ \text{ }=\frac{49}{8}\hfill \end{cases}[/latex] The following video provides another example of finding the average rate of change of a function given a formula and an interval. Find the average rate of change of [latex]f\left(x\right)=x - 2\sqrt{x}[/latex] on the interval [latex]\left[1,9\right][/latex]. Solution The electrostatic force [latex]F[/latex], measured in newtons, between two charged particles can be related to the distance between the particles [latex]d[/latex], in centimeters, by the formula [latex]F\left(d\right)=\frac{2}{{d}^{2}}[/latex]. Find the average rate of change of force if the distance between the particles is increased from 2 cm to 6 cm. We are computing the average rate of change of [latex]F\left(d\right)=\frac{2}{{d}^{2}}[/latex] on the interval [latex]\left[2,6\right][/latex]. [latex]\begin{cases}\text{Average rate of change }=\frac{F\left(6\right)-F\left(2\right)}{6 - 2}\\ {}\\ =\frac{\frac{2}{{6}^{2}}-\frac{2}{{2}^{2}}}{6 - 2} & \text{Simplify}. \\ {}\\=\frac{\frac{2}{36}-\frac{2}{4}}{4}\\{}\\ =\frac{-\frac{16}{36}}{4}\text{Combine numerator terms}.\\ {}\\=-\frac{1}{9}\text{Simplify}\end{cases}[/latex] The average rate of change is [latex]-\frac{1}{9}[/latex] newton per centimeter. Find the average rate of change of [latex]g\left(t\right)={t}^{2}+3t+1[/latex] on the interval [latex]\left[0,a\right][/latex]. The answer will be an expression involving [latex]a[/latex]. We use the average rate of change formula. [latex]\text{Average rate of change}=\frac{g\left(a\right)-g\left(0\right)}{a - 0}\text{Evaluate}[/latex]. = [latex]\frac{\left({a}^{2}+3a+1\right)-\left({0}^{2}+3\left(0\right)+1\right)}{a - 0}\text{Simplify}.[/latex] =[latex]\frac{{a}^{2}+3a+1 - 1}{a}\text{Simplify and factor}.[/latex] =[latex]\frac{a\left(a+3\right)}{a}\text{Divide by the common factor }a.[/latex] =[latex]a+3[/latex] This result tells us the average rate of change in terms of [latex]a[/latex] between [latex]t=0[/latex] and any other point [latex]t=a[/latex]. For example, on the interval [latex]\left[0,5\right][/latex], the average rate of change would be [latex]5+3=8[/latex]. Find the average rate of change of [latex]f\left(x\right)={x}^{2}+2x - 8[/latex] on the interval [latex]\left[5,a\right][/latex]. Solution blovy Published in Calculations 3 mins read Apr 21, 2025 The average change (also known as the average rate of change) is calculated using the following formula: (y2 - y1) / (x2 - x1), where (x1, y1) and (x2, y2) are two points. This formula essentially calculates the slope of the line connecting the two points. Understanding Average Change Average change measures how much a quantity changes on average over a specific interval. It's a fundamental concept with applications in various fields, including: Mathematics: Determining the slope of a secant line. Finance: Calculating the average growth rate of investments. Physics: Finding the average velocity of an object. Economics: Measuring the average change in prices or production. Formula Breakdown Let's break down the formula for average change: y2 - y1: This represents the change in the y-value (dependent variable). It's often referred to as "rise." x2 - x1: This represents the change in the x-value (independent variable). It's often referred to as "run." (y2 - y1) / (x2 - x1): This division calculates the rate at which the y-value changes per unit of change in the x-value. Steps for Calculating Average Change Identify two points: You need two data points represented as coordinates (x1, y1) and (x2, y2). These points could be given directly or extracted from a graph or table. Calculate the change in y: Subtract the initial y-value (y1) from the final y-value (y2). This gives you y2 - y1. Calculate the change in x: Subtract the initial x-value (x1) from the final x-value (x2). This gives you x2 - x1. Divide the change in y by the change in x: Divide the result from step 2 by the result from step 3. This is the average rate of change: (y2 - y1) / (x2 - x1). Example Let's say you want to find the average change in temperature between 8:00 AM and 12:00 PM. At 8:00 AM (x1), the temperature (y1) is 60°F. At 12:00 PM (x2), the temperature (y2) is 72°F. Using the formula: Average change = (72 - 60) / (12 - 8) = 12 / 4 = 3°F per hour. This means that, on average, the temperature increased by 3 degrees Fahrenheit each hour between 8:00 AM and 12:00 PM. Important Considerations Units: Always include the appropriate units for the average rate of change. In the example above, the units are degrees Fahrenheit per hour. Linearity: The average rate of change assumes a linear relationship between the two points. It doesn't necessarily reflect the exact change at every point within the interval. For a non-linear relationship, the instantaneous rate of change (using calculus) provides a more precise measure at a specific point. Context: Interpret the average rate of change within the context of the problem. Understand what the x and y values represent to make meaningful conclusions. In mathematics and real-life applications, the concept of rate of change plays a significant role in analyzing the development or behavior of various trends. Particularly, the average rate of change is an important measurement that allows us to determine how a particular variable is changing with respect to another variable over a given interval. This article will guide you through the process of calculating the average rate of change step by step. Step 1: Understand the Concept Before diving into calculations, it's essential to understand what the average rate of change represents. It signifies how one variable changes (usually denoted as y) in relation to another variable (usually denoted as x) over a particular interval. In simple terms, it's like finding the slope of a straight line that connects two points on a curve. Step 2: Identify the Variables The first step to calculate the average rate of change is identifying values for x (the independent variable) and corresponding values for y (the dependent variable). Generally, these values are provided within a question or can be collected from real-life data. Step 3: Determine the Interval Identify which interval on which you need to calculate the average rate of change. The interval is typically provided in problems involving average rate of change but can also be chosen based on your requirement for specific data analysis. Step 4: Calculate Differences For each pair of x-values within your chosen interval, determine the difference in their corresponding y-values. To do this, subtract the initial y-value from the final y-value. Change in y = Final y – Initial y Next, calculate the difference in x-values by subtracting the initial x-value from the final x-value. Change in x = Final x – Initial x Step 5: Applying Formula Now that you have all required information from previous steps, applying it using this formula: Average rate of change = Change in y / Change in x Ensure you have the correct signs when performing the division, as it will impact the final result, indicating whether your average rate of change is positive, negative, or equal to zero. Step 6: Interpret the Result Finally, interpret your result to understand the significance of the average rate of change. A positive value indicates an increase in 'y' as 'x' increases, while a negative value signifies a decrease in 'y' as 'x' increases. If the average rate of change is 0, it indicates that there is no change in 'y' concerning 'x.' Conclusion Calculating the average rate of change is an essential skill in various fields such as mathematics, economics, physics, and data analytics. By following these six easy steps, you can determine the average rate of change over a given interval and interpret its meaning for greater insights into trends and data relationships. The average rate of change describes the average rate at which one quantity is changing with respect to another. It gives an idea of how much the function changed per unit in the given interval. In simple terms, in the rate of change, the amount of change in one item is divided by the corresponding amount of change in another. Let's look into the average rate of change formula in detail. What is the Average Rate of Change? The average rate of change of a function f(x) over an interval [a, b] is defined as the ratio of "change in the function values" to the "change in the endpoints of the interval". i.e., the average rate of change can be calculated using [f(b) - f(a)] / (b - a). In other words, the average rate of change (which is denoted by A(x)) is the "ratio of change in outputs to change in inputs". i.e., A(x) = (change in outputs) / (change in inputs) = Δy / Δx = [f(b) - f(a)] / (b - a) Here, Δy is the change in y-values (or) change in the function values and Δx is the change in x-values (or) the change in the endpoints of the interval. Some examples of the average rate of change are: A bus travels at a speed of 80 km per hour. The number of fish in a lake increases at the rate of 100 per week. The current in an electrical circuit decreases 0.2 amperes for a decrease of 1-volt voltage. Average Rate of Change Formula The average rate of change function describes the average rate at which one quantity is changing with respect to something another quantity. The average rate of change formula is given as, A(x) = [f(b) - f(a)] / (b - a) where, A(x) = Average rate of change f(a) = Value of function f(x) at a f(b) = Value of function f(x) at b Have questions on basic mathematical concepts? Become a problem-solving champ using logic, not rules. Learn the why behind math with our certified experts Book a free Trial Class Examples Using Average Rate of Change Formula Example 1: Calculate the average rate of change of a function, f(x) = 2x + 10 as x changes from 3 to 7. Solution: Given: f(x) = 2x + 10, a = 3, b = 7. f(3) = 2(3) + 10 = 6 + 10 = 16 f(7) = 2(7) + 10 = 14 + 10 = 24 Using the average rate of change formula, A(x) = [f(b)−f(a)] / (b−a) A(x) = [f(7)−f(3)]/ (7−3) A(x) = (24 - 16) / 4 A(x) = 8/4 A(x) = 2 Therefore, the rate of change is 2. Example 2: Evaluate the average rate of change of the function f(x) = x2 – 5x in the interval 4 ≤ x ≤ 8. Solution: Given: f(x) = x2 – 5x, a = 4, b = 8. f(4) = f(4) = (4)2 - 5(4) = 16 - 20 = -4 f(8) = (8)2 - 5(8) = 64 - 40 = 24 Using the average rate of change formula, A(x) = [f(b)−f(a)] / (b−a) = (24−(−4)) / (8−4) = 28/4 = 7 Therefore, rate of change A(x) = 7. Example 3: Find the rate of change of a function, f(x) = 25x + 18 as x changes from 5 to 8 Solution: Given, f(x) = 25x + 18, a = 5 and b = 8 f(5) = 25×5 + 18 = 143 f(8) = 25×8 + 18 = 218 Using the average rate of change formula, A(x) = [f(b)−f(a)] / (b−a) = [218 - 143] / (8 - 5) = 75 / 3 = 25 Therefore, the average rate of change A(x) = 25 The average rate of change is the change one quantity with respect to the change in another. It is a measure of how much the function changed per unit in a particular interval. If f(x) is the function and [a, b] is the interval, then the formula is A(x) = [f(b) - f(a)] / (b - a) What is the Formula to Find the Average Rate of Change? The average rate of change formula is given as, A(x) = [f(b) - f(a)] / (b - a) where, A(x) = Average rate of change f(a) = Value of function f(x) at a f(b) = Value of function f(x) at b How to Find Average Rate of Change Over an Interval? To find the average rate of change of a function f(x) over an interval [a, b]: Find f(a) and f(b). Substitute the values in the formula [f(b) - f(a)] / (b - a) and simplify. Note that it can be calculated using the formula [f(a) - f(b)] / (a - b) as well. But make sure to follow the same order both in the numerator and the denominator. What is the Formula to Find the Rate of Change of a Linear Function? For a linear function, the rate of change is represented by the parameter (m) in the slope-intercept form for a line: y=mx+b, and is visible in a table or on a graph. Is the Average Rate of Change the Same as Slope? The slope is considered as the average rate of change of a point where the average is taken and is reduced to zero. The slope is the rise over the run which is defined as the average rate of change in y coordinates over the change in x coordinates.